23 in 4.1

$$f(x) = \begin{cases} \frac{5}{2} - x & ; x < 2 \\ \frac{1}{x} & ; x \geq 2 \end{cases}$$

find $f^{-1}(x)$

4.1) 28 or 29
4.1) 23
4.1) 32
4.3) 3

Piece 1

$g(x) = \frac{5}{2} - x$

Domain
$x < 2$

Range
$(\frac{1}{2}, \infty)$

$y = \frac{5}{2} - x$

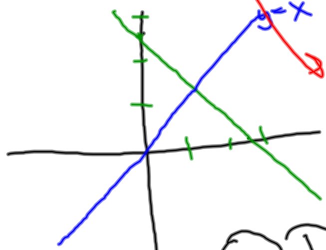
$x = \frac{5}{2} - y$

$y = \frac{5}{2} - x$

$f^{-1}(x) = \frac{5}{2} - x$

Domain of f^{-1}
$(\frac{1}{2}, \infty)$

Range of f^{-1}
$(-\infty, 2)$



Piece 2

$h(x) = \frac{1}{x}$

$\frac{1}{x} = 0$? No nonsense

$y = \frac{1}{x}$

$\Rightarrow$

$x = \frac{1}{y}$

$\Rightarrow$

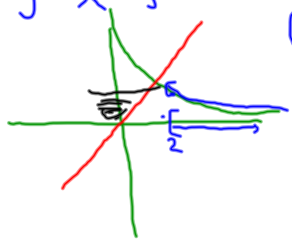
$y = \frac{1}{x} = f^{-1}(x)$

Domain of f
$[2, \infty)$

Range of f
$(0, \frac{1}{2}]$

Domain of f^{-1}
$(0, \frac{1}{2}]$

Range of f^{-1}
$[2, \infty)$



ORTHOGONAL

$$f^{-1}(x) = \begin{cases} \frac{5}{2} - x, & x \in (\frac{1}{2}, \infty) \\ \frac{1}{x}, & x \in (0, \frac{1}{2}] \end{cases}$$

$0 < x \leq \frac{1}{2}$

4.1 28) $f(x) = 3x^2 + 5x - 2$; $x \geq 0$

range: $y \geq -2$

rule

$3x^2 + 5x + 100$

$3x^2 + 5x - \frac{7}{3}$

$y = 3x^2 + 5x - 2$

$3x^2 + 5x = x(3x + 5)$

$x = 3y^2 + 5y - 2$

$x = 0$

$x = -\frac{5}{3}$

$x = 3\left(y^2 + \frac{5}{3}y\right) - 2$

$x + 2 = 3\left(y^2 + \frac{5}{3}y\right)$

$x + 2 = 3\left(\left(y + \frac{5}{6}\right)^2 - \frac{25}{36}\right)$

$y^2 + \frac{5}{6}y + \frac{5}{6}y + \frac{25}{36}$

$\frac{x+2}{3} = \left(y + \frac{5}{6}\right)^2 - \frac{25}{36}$

$\frac{x+2}{3} + \frac{25}{36} = \left(y + \frac{5}{6}\right)^2$

$y + \frac{5}{6} = \sqrt{\frac{x+2}{3} + \frac{25}{36}}$

$f^{-1} = \sqrt{\frac{x+2}{3} + \frac{25}{36}} - \frac{5}{6}$ $x \geq -2$

$y + \frac{5}{6} = -\sqrt{\frac{x+2}{3} + \frac{25}{36}}$

handout $f(x) = x^3 + 2x - 1 \; ; x = 2 \implies$ $x = 2$, $y = 8 + 4 - 1 = 11$

$$\left[f^{-1}(x)\right]' = \frac{1}{f'(f^{-1}(x))}$$

$$\left[f^{-1}(11)\right]' = \frac{1}{f'(f^{-1}(11))} = \frac{1}{f'(2)} = \frac{1}{14}$$

$$f'(x) = 3x^2 + 2$$

$$f'(2) = 3(2)^2 + 2 = 14$$

Method 2

$$x = y^3 + 2y - 1$$

$$1 = 3y^2 \frac{dy}{dx} + 2\frac{dy}{dx}$$

$$1 = \frac{dy}{dx}(3y^2 + 2)$$

$$\frac{1}{3y^2 + 2} = \frac{dy}{dx}$$

$$\frac{1}{3(2)^2 + 2} = \left.\frac{dy}{dx}\right|_{y=2}$$

$$= \frac{1}{14}$$

4.2 32

$$e^{2x} - e^{x} = 6$$

$$e^{2x} = (e^{x})^{2}$$

Let $u = e^{x}$

$$u^2 - u = 6$$

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u = -2, +3$$

$\Rightarrow e^{x} = -2$ or $e^{x} = 3$

$\neq$ Not possible

$$\ln(e^{x}) = \ln 3$$

$$x = \ln 3$$

4.3) 3 $y=(\ln x)^2$

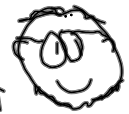
$2(\ln x)\cdot\left(\frac{1}{x}\right)$

$y' = \frac{2\ln x}{x}$

$f(x)=x^2 \ldots\ldots f'(x)=2x$

$g(x)=\ln x \ldots\ldots g'(x)=\frac{1}{x}$

$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$



$$\int \frac{2\ln x}{x}\,dx = (\ln x)^2 + C$$

$$\int \frac{\ln x}{x}\,dx =$$

Let $u=\ln x$

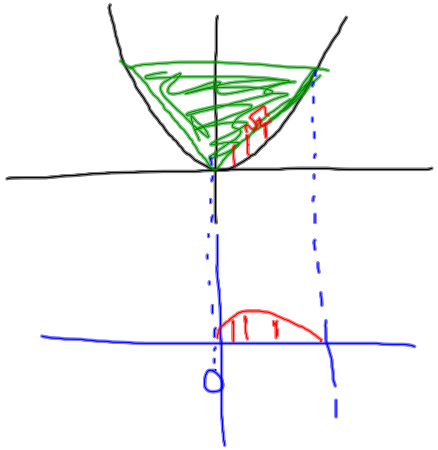
$\frac{du}{dx}=\frac{1}{x}$

$dx\cdot\frac{du}{dx} = \ln x\cdot dx$

$du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{u^2}{2}+C$$

$$\frac{(\ln x)^2}{2}+C$$



$D_x\, f \cdot g = f' g + f g'$

23
25
18
21
29

16) $y = \overset{f}{x}\,\overbrace{\left[\log_2(x^2-2x)\right]^3}^{g}$. Find y'

$$y' = \frac{d}{dx}(x)\left[\log_2(x^2-2x)\right]^3 + x\frac{d}{dx}\left[\log_2(x^2-2x)\right]^3$$

$$= \left[\log_2(x^2-2x)\right]^3 + x\left[3\left[\log_2(x^2-2x)\right]^2 \cdot \frac{d}{dx}\left[\log_2(x^2-2x)\right]\right]$$

$$= \left[\log_2(x^2-2x)\right]^3 + 3x\left[\log_2(x^2-2x)\right]^2\left[\frac{1}{(\ln 2)(x^2-2x)} \cdot \frac{d}{dx}(x^2-2x)\right]$$

P 382

$$\frac{d}{dx}\log_b(x) = \frac{1}{(\ln b)x}$$

$$= \left[\log_2(x^2-2x)\right]^3 + \frac{3x\left[\log_2(x^2-2x)\right]^2}{(\ln 2)(x^2-2x)}(2x-2)$$

18) $y = \dfrac{\log_{10} x}{1 + \log_{10} x}$

$$\frac{d}{dx}\left(\frac{\text{Top}}{\text{Bottom}}\right) = \frac{(\text{Top})'\text{Bottom} - (\text{Top})(\text{Bottom})'}{(\text{Bottom})^2}$$

p.380
$\frac{d}{dx}(\log_b x) = \frac{1}{(\ln b)x}$

$$y' = \frac{\frac{d}{dx}(\log x)\cdot(1+\log x) - (\log x)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$= \frac{\left(\frac{1}{(\ln 10)x}\right)(1+\log x) - \frac{\log x}{(\ln 10)x}}{(1+\log x)^2}$$

$$= \frac{1}{(\ln 10)(x)(1+\log x)^2}$$

Find $\int \frac{1}{x(1+\log x)^2}\,dx$ by u-substitution

$u = 1 + \log x$

$\frac{du}{dx} = \frac{1}{(\ln 10)x}$

$du = \frac{1}{(\ln 10)x}\,dx$

$(\ln 10)\,du = \frac{1}{x}\,dx$

$(\ln 10)\int \frac{1}{u^2}\,du = \ln 10 \int$

$= (\ln 10)\,\frac{u^{-1}}{-1} + C$

$= -\frac{\ln 10}{u} + C$

$= \frac{-\ln 10}{(1+\log x)} + C$

$\frac{\log_{10} x}{1+\log_{10} x}$? $\frac{-1}{(1+\log x)} + C$

$$\frac{\log x}{1+\log x} = \frac{1+\log x}{1+\log x} - \frac{1}{1+\log x}$$

Inv Packet # 2

$$f(x) = 2x^5 + x^3 + 1 \qquad x = y$$

method 2

$$x = 4 \Rightarrow f(x) = 2\cdot 4^5 + 4^3 + 1 = 2113$$

$$x = 2y^5 + y^3 + 1$$

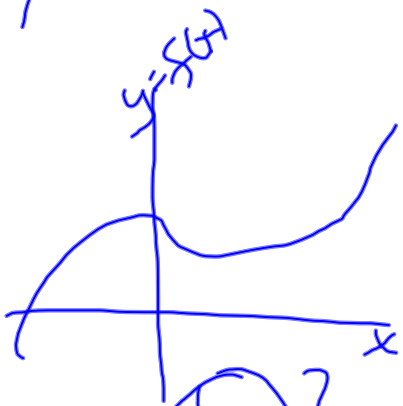
take the derivative with respect to x.

$$1 = 10y^4 \frac{dy}{dx} + 3y^2 \frac{dy}{dx}$$

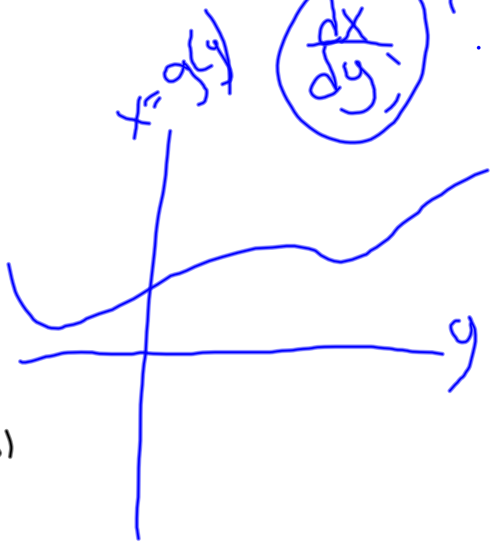
$$1 = \frac{dy}{dx}(10y^4 + 3y^2)$$

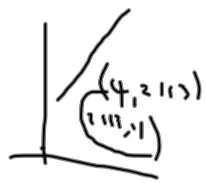
$$\frac{1}{10y^4 + 3y^2} = \frac{dy}{dx}$$

$$\frac{1}{10\cdot 4^4 + 3\cdot 4^2} = \frac{dy}{dx}$$



$\frac{dx}{dy}$?





28) $y = \exp\left(\sqrt{1+5x^3}\right) = e^{\sqrt{1+5x^3}}$

$$y' = e^{\sqrt{1+5x^3}} \underbrace{\left(\tfrac{1}{2}(1+5x^3)^{-\frac{1}{2}}\right)}_{\frac{d}{dx}\sqrt{1+5x^3}} \underbrace{\left(15x^2\right)}_{\frac{d}{dx}(5x^3)}$$

21) $y = x^3 e^x$

4.1/29 $f(x) = x - 5x^2$; $x \geq 1$

4.1/28 or 29
4.2/34
4.3/9 16?
14
12

Dom: $[1, \infty)$

Range: $(-\infty, -4]$

$y = x - 5x^2$

$x = y - 5y^2$

$-x = 5y^2 - y$

$-x = 5(y^2 - \frac{1}{5}y \quad)$

$-x = 5\left[\left(y - \frac{1}{10}\right)^2 - \frac{1}{100}\right]$

$y^2 - \frac{1}{10}y - \frac{1}{10}y + \frac{1}{100}$

$x - 5x^2 = 0$

$x(1 - 5x) = 0 \Rightarrow x = 0, \frac{1}{5}$

$\frac{1}{10}$

Domain of $f^{-1}(x)$ = Range of $f(x)$: $(-\infty, -4]$

$\frac{-x}{5} = \left(y - \frac{1}{10}\right)^2 - \frac{1}{100}$

$\frac{-x}{5} + \frac{1}{100} = \left(y - \frac{1}{10}\right)^2$

$y - \frac{1}{10} = \sqrt{\frac{1}{100} - \frac{x}{5}}$

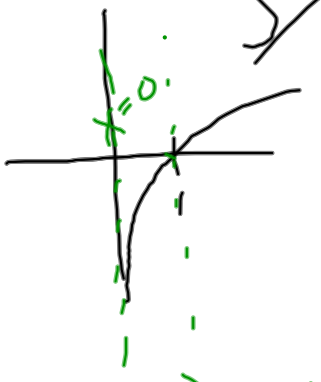
$y - \frac{1}{10} = -\sqrt{\frac{1}{100} - \frac{x}{5}}$

$y = f^{-1}(x) = \frac{1}{10} - \sqrt{\frac{1}{100} - \frac{x}{5}}$

4.2 | 34

sketch $y = 1 + \ln(x-2)$

move 2 to the right

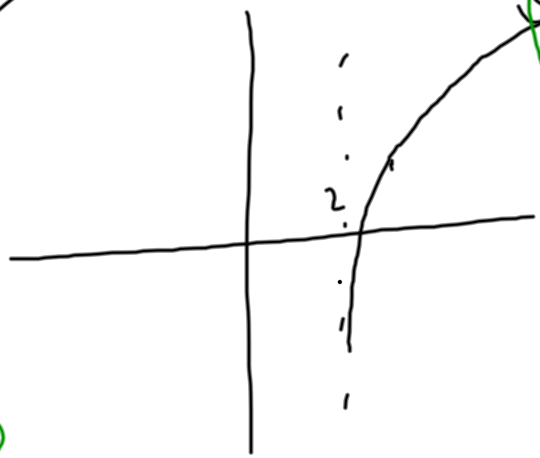


$(x-2) = 0$

$x = 2$ VA.

$(x-2) = 1$

$x = 3$ zero



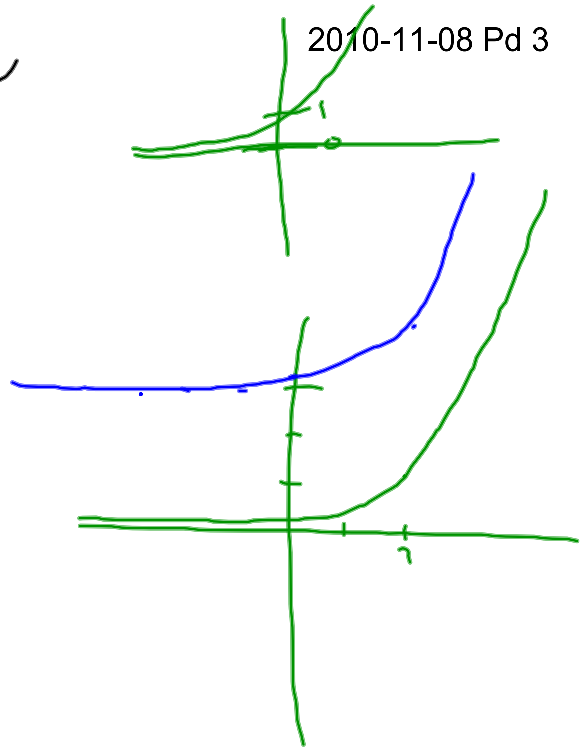
$0 = 1 + \ln(x-2)$

$e^{(-1)} = e^{(\ln(x-2))}$

$e^{-1} = x - 2$

$\frac{1}{e} + 2 = x$

$y = 3 + e^{x-2}$



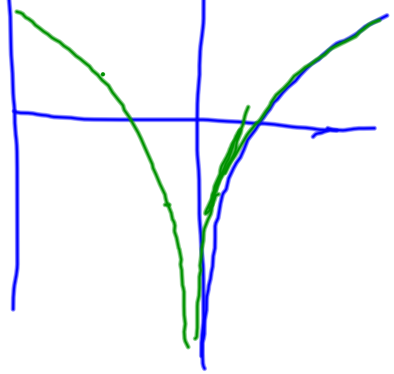
4.3) 9 $y = \ln(|x^3 - 7x^2 - 3|)$

$$y' = \frac{1}{x^3-7x^2-3} \frac{d}{dx}(|x^3-7x^2-3|)$$

$$= \frac{1}{x^3-7x^2-3}(3x^2-14x)\left(\frac{|x^3-7x^2-3|}{x^3-7x^2-3}\right)$$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \ln|x| = \frac{1}{x}$



$\frac{d}{dx}(|x|) =$

$\left(\frac{|x|}{x}\right)$

10) $y = x^3 \ln x$

$y' = 3x^2 \ln x + x^3\left(\frac{1}{x}\right)$

$= 3x^2 \ln x + x^2$

$= x^2(3\ln x + 1)$

12) $y = \sqrt{1 + \ln^2 x}$

$y = (1 + \ln^2 x)^{1/2}$

$y' = \frac{1}{2}(1 + \ln^2 x)^{-1/2} \cdot \frac{d}{dx}(1 + \ln^2 x)$

$y' = \frac{1}{2}(1 + \ln^2 x)^{-1/2} \cdot (2 \ln x)\left(\frac{d}{dx} \ln x\right)$

$y' = \frac{1}{2}(1 + \ln^2 x)^{-1/2} \cdot 2 \ln x \left(\frac{1}{x}\right)$

14) $y = \sin^2(\ln x)$

$= [\sin(\ln x)]^2$

$y' = 2[\sin(\ln x)](\cos(\ln x))\left(\frac{1}{x}\right)$

4.3 homework

2010-11-08 Pd 3

6) $y = \ln(2+\sqrt{x})$

$$y' = \frac{1}{2+\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)$$

8) $y = \ln(\ln x)$

$$y' = \frac{1}{\ln x}\left(\frac{1}{x}\right)$$

$$\int \frac{1}{x\ln x}\,dx = \ln(\ln(x)) + C$$

$$\int \frac{1}{x\ln x}\,dx$$

$u = $ $\frac{\ln x}{x \ln x}$

$(\ln x)^{-1} = \frac{1}{\ln x}$

Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int \frac{1}{u}\,du$$

$$= \ln|u| + C = \ln|\ln x| + C$$

$$\int \frac{1}{x\ln x}\,dx$$

Let $u = x\ln x$

$\frac{du}{dx} = (1)\ln x + \frac{x}{x}$

$= 1 + \ln x$

$du = (1+\ln x)\,dx$

Probably too much trouble

$$\int \frac{x}{x^2 \ln x}\,dx$$

Let $u = x^{-1}$

$du = \frac{-1}{x^2}dx$

$$-\int \frac{1}{u}\,du$$

probably not the best

23
3.7

29) $y = \ln(1 - xe^{-x})$

$-\frac{d}{dx}(xe^{-x})$

$\frac{d}{dx}(-xe^{-x})$

$(-x)\cdot(e^{-x})$

$$y' = \frac{1}{(1-xe^{-x})}\left(\frac{d}{dx}(1-xe^{-x})\right)$$

$$= \frac{1}{1-xe^{-x}}\left(-\left[\frac{d}{dx}(x)\cdot e^{-x} + x\cdot\frac{d}{dx}(e^{-x})\right]\right)$$

$$= \frac{1}{1-xe^{-x}}\left(-\left[e^{-x} + x\left(e^{-x}\frac{d}{dx}(-x)\right)\right]\right)$$

$$= \frac{1}{1-xe^{-x}}\left(-\left[e^{-x} + (x)(e^{-x})(-1)\right]\right)$$

$$= \frac{1}{1-xe^{-x}}\left(-e^{-x} + xe^{-x}\right)$$

$$= \frac{-1}{1-\frac{x}{e^x}}\left(\frac{1}{e^x} - \frac{x}{e^x}\right) = \frac{-1}{1-\frac{x}{e^x}}\left(\frac{-x+1}{e^x}\right)$$

$$= \frac{-(-x+1)}{e^x\left(1-\frac{x}{e^x}\right)} = +\frac{(x-1)}{e^x - x}$$

$$= \frac{-1+x}{e^x - x}$$

$$\frac{x-1}{e^x - x}$$

23)

$$y = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$ hyperbolic tangent

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$y' = \frac{\frac{d}{dx}(e^{x} - e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})\frac{d}{dx}(e^{x} + e^{-x})}{(e^{x} + e^{-x})^2}$$

$$= \frac{\overbrace{(e^{x} - e^{-x}(-1))}^{e^x + e^{-x}}(e^{x} + e^{-x}) - (e^{x} - e^{-x})\overbrace{(e^{x} + e^{-x}(-1))}^{e^x - e^{-x}}}{(e^{x} + e^{-x})^2}$$

$(e^{x})^2$
$= e^{2x}$

$$= \frac{(e^{x} + e^{-x})^2 - (e^{x} - e^{-x})^2}{(e^{x} + e^{-x})^2}$$

catenary curve

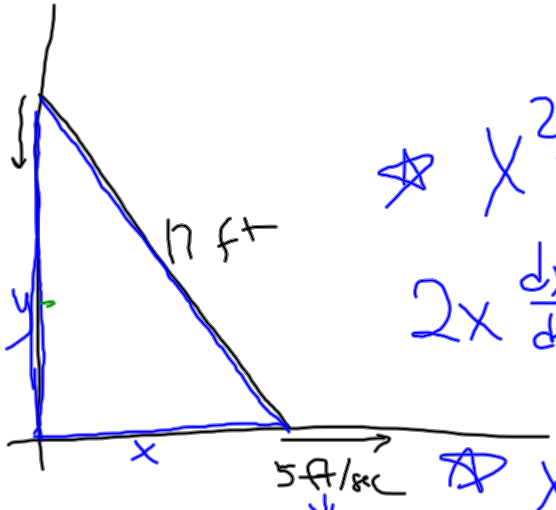
$e^{x} \cdot e^{-x}$
$= e^{(x + -x)}$
$= e^{0}$

$$= \frac{e^{2x} + 1 + 1 + e^{-2x} - (e^{2x} - 1 - 1 + e^{-2x})}{(e^{x} + e^{-x})^2}$$

$$= \frac{4}{(e^{x} + e^{-x})^2}$$

$$= \left(\frac{2}{e^{x} + e^{-x}}\right)^2$$

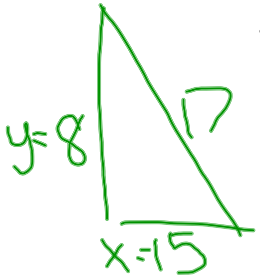
related rates 16



$$x^2+y^2=17^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$



$$15(5)+8\frac{dy}{dt}=0$$

$$\frac{-75}{8}=\frac{dy}{dt}$$

ft/sec

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$\sqrt{289-81}$

$=\sqrt{208}$

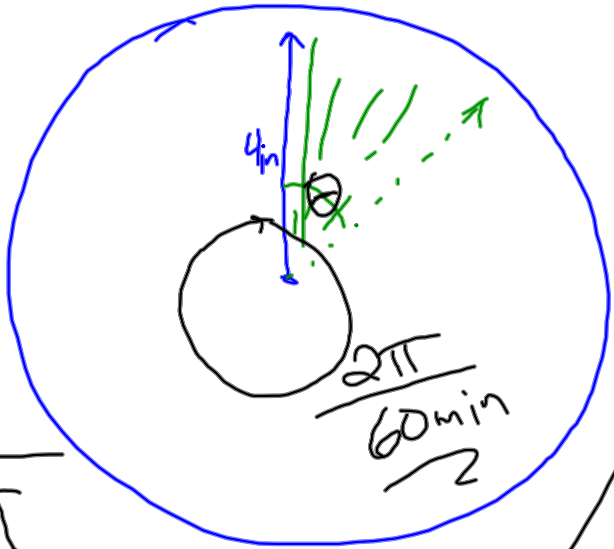
$\sqrt{289-36}=\sqrt{253}$

$$\sqrt{208}(5)+9\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=\frac{-5\sqrt{208}}{9}$$

$$\frac{dy}{dt}=\frac{-5\sqrt{253}}{6}$$

rel rates 11



$$\frac{\text{Area}_{\text{sector}}}{\text{Area}_{\text{circle}}} = \frac{\theta}{2\pi}$$

$$\text{Area}_{\text{sector}} = \frac{\theta}{2\pi}(\pi r^2)$$

$$\checkmark\ \boxed{A = \frac{\theta}{2} r^2} = 8\theta$$

$$\checkmark\ A = 8\theta$$

$$\boxed{\frac{dA}{dt} = 8\frac{d\theta}{dt}}$$

$$\frac{dA}{dt} = 8\left(\frac{2\pi}{60}\right)$$

$$= \frac{4\pi}{15}\ \text{in}^2/\text{min}$$

21) $y = x^3 e^x$ $\qquad \frac{d}{dx}(f \cdot g) = f'g + fg'$

$$y' = \frac{d}{dx}(x^3) \cdot e^x + x^3 \cdot \frac{d}{dx}(e^x)$$

$$= (3x^2)e^x + x^3(e^x)$$

$$= x^2 e^x (3 + x) = 0$$

$x = 0$ $\qquad x = -3$

4.3 homework

2010-11-09 Pd 3